Course Syllabus

I. General Information

Course name	Measure and integral
Programme	Mathematics
Level of studies (BA, BSc, MA, MSc, long-cycle MA)	BA
Form of studies (full-time, part-time)	Full-time
Discipline	Mathematics
Language of instruction	English

Course coordinator	dr hab. August Zapała

Type of class *(use only the types mentioned below)*	Number of teaching hours	Semester	ECTS Points
lecture	30	IV	5
tutorial			
classes	30	IV	
laboratory classes			
workshops			
seminar			
introductory seminar			
foreign language classes			
practical placement			
field work			
diploma laboratory			
translation classes			
study visit			

Course pre-requisites	Introduction to mathematics, Calculus I, Calculus II, Topology

II. Course Objectives

Students gain basic knowledge concerning general measure theory
Students gain knowledge concerning Lebesgue measure
Students gain knowledge concerning Lebesgue integral

III. Course learning outcomes with reference to programme learning outcomes

Symbol	Description of course learning outcome	Reference to programme learning outcome
	KNOWLEDGE	
W_01	Students understand the construction of the theory of measure, learn the general construction of the measure using an external measure	K_W01, K_W03
W_02	Students learn the most important definitions and theorems of the theory of measure and integral	K_W02, K_W04, K_W07
W_03	Students learn the construction of the Lebesgue measure and its properties	K_W03, K_W04, K_W05
W_04	Students learn the construction of the integral relative to any measure	K_W03, K_W04
	SKILLS	
U_01	Students can calculate the Lebesgue measure of various sets	K_U01, K_U02, K_U04, K_U09
U_02	Students can calculate Lebesgue integrals of various measurable functions	K_U02, K_U04, K_U13
U_03	Students are able to use the Lebesgue measure and integral in various theoretical and practical problems	K_U03, K_U05, K_U06, K_U07, K_U14
	SOCIAL COMPETENCIES	
K_01	Students precisely formulate questions to deepen the understanding of the subject and complement the missing elements of reasoning	K_K01
K_02	Students present opinions on the applicability of measure and integral theory taking into account own knowledge and skills	K_K01,K_K05

IV. Course Content

The notions of a field and σ-field of sets. Lemma on existence of the minimal σ-field of sets generated by a given class of sets.
Definition of measure. General properties of measures (monotonicity, finiteness and σ-finiteness, subadditivity, asymptotic properties on monotone sequences of sets). Sets of measure zero and the theorem concerning completion of a measure.
The measurability condition and Caratheodory's lemma.
Outer measure and the Caratheodory theorem on extension of outer measure to a measure.
Theorem concerning an extension of a measure defined on the field to a σ-field.
Outer Lebesgue measure. Construction of the Lebesgue measure.
Relationships of the Lebesgue measure with the volume of sets in the Euclidean space.
Outer metric measure in a metric space and measurability of Borel sets.
Measurability of Borel sets in the sense of Lebesgue. Cantor ternary set and a theorem on existence of non-Borel sets measurable in the Lebesgue sense.
De la Vallée-Poussin conditions for measurability of sets in the sense of Lebesgue.
Shift invariance of outer Lebesgue measure and Lebesgue measure.
Construction of a nonmeasurable set in the sense of Lebesgue.
The notion of a measurable function with respect to the σ-field of sets.
Measurability of mappings obtained by means of various algebraic operations on measurable functions. Sequences of measurable functions, measurability of supremum, infimum and limits of sequences of measurable functions. Fundamental properties of simple functions related to measurability. Theorem on approximation of measurable functions by means of simple functions.

Integrals of simple functions and their basic properties (positivity, positive homogeneity, linearity, monotonicity, additivity of the integral as a set function). Basic lemmas on integration of simple functions. Construction of the Lebesgue integral. The most important properties of the Lebesgue integral (e.g. integrals of functions equal a.e., finiteness a.e. of the integrable function, linearity, additivity of the integral as a set function, etc.). Criteria for integrability of functions in the Lebesgue sense. Convergence a.e. and its basic properties. Convergence in measure and its properties. Connections between various types of convergence. Beppo-Levi theorem. Lebesgue's theorems on integration of monotone sequnces and series of functions. Fatou's lemma. The Lebesgue dominated convergence theorems. Information concerning spaces of integrable functions.

V. Didactic methods used and forms of assessment of learning outcomes

Symbol	Didactic methods *(choose from the list)*	Forms of assessment *(choose from the list)*	Documentation type *(choose from the list)*
	KNOWLEDGE		
W_01	conventional lecture, discussion, practical classes	test, written exam, oral exam	evaluated test, protocol
W_02	conventional lecture, discussion, practical classes	test, written exam, oral exam	evaluated test, protocol
W_03	conventional lecture, discussion, practical classes	test, written exam, oral exam	evaluated test, protocol
W_04	conventional lecture, discussion, practical classes	test, written exam, oral exam	evaluated test, protocol
	SKILLS		
U_01	conventional lecture, discussion, practical classes, laboratory classes	test, written exam, oral exam	evaluated test, protocol
U_02	conventional lecture, discussion, practical classes, laboratory classes	test, written exam, oral exam	evaluated test, protocol
U_03	conventional lecture, discussion, practical classes, laboratory classes	test, written exam, oral exam	evaluated test, protocol
	SOCIAL COMPETENCIES		
K_01	conventional lecture, discussion, practical classes, laboratory classes, problem-based learning	test, written exam, oral exam	evaluated test, protocol
K_02	conventional lecture, discussion, practical classes, laboratory classes, problem-based learning	test, written exam, oral exam	evaluated test, protocol

VI. Grading criteria, weighting factors.....

LECTURE:
The completion of classes is required. Written exam constitute the final grade:
91 – 100% excellent
81 – 90% very good
71 – 80% good
61 – 70% satisfactory
51 – 60% sufficient
less than 51% fail
CLASSES:
At least 80% of attendance is required. Two tests together constitute the final grade:
91 – 100% excellent
81 – 90% very good
71 – 80% good
61 – 70% satisfactory
51 – 60% sufficient
less than 51% fail
Detailed assessment rules are given during lectures and classes.

VII. Student workload

Form of activity	Number of hours
Number of contact hours (with the teacher)	**90**
Number of hours of individual student work	**60**

VIII. Literature

Basic literature
R. Sikorski, Funkcje rzeczywiste, t. I, PWN 1958 S. Łojasiewicz, Wstęp do teorii funkcji rzeczywistych, PWN 1976 R. L. Schilling, Measures, Integrals and Martingales, Cambridge 2005
Additional literature
J. Niewiarowski, Zadania z teorii miary